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MMWD
Making Migration Work for Development

Programme co-funded by the EUROPEAN UNION

METHODOLOGICAL REFERENCES FOR POPULATION PROJECTIONS

1. Operational Overview

These methodological references are intended as a tool for the preparation, by the experts selected and appointed by the Project Partners, of demographic projections whose first results and findings are to be compulsorily submitted by all Project Partners in the MMWD Scientific and Steering Committee of Bucharest on the 6th and 7th of November, 2013.

These methodological references are based on the report prepared by the ISSK-BAS on MMWD consortium's similarities and differences in the methodologies for population projections. The report analysed the information provided by MMWD consortium partners on the established methods for national and regional demographic forecasts and highlighted Project Partners differences in terms of methodologies and methods, sources of information, geographic districts and periods covered, definitions, etc., in population projections.

These references are operating the result of a comparison process culminated with the Expert Meeting of the 19th and the 20th of September in Bologna. References are partly a compromise between homogeneity and heterogeneity of projections methods adopted in the MMWD consortium, definitions and assumptions, but also take into account the availability and unavailability of data.

The demographic experts should approximate as much as possible these references and, in the case of substantial deviations, inform and / or deal with the following contact persons: .

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2. Basic Population Projections

The basic population projections are:

- Projection by age/sex/total population
- Projection by age/sex/foreign population

These are therefore strictly demographic projections that provide the amount and structure of the population.

Foreign population, if age and sex profiles are available by citizenship, should be split at least in EU/non EU, according to the recommendations made during 2010 Eurostat's Work session on demographic projections[1].

The basic population projections have to cover the period between the last available data and 2020.

Basic population projections should be transmitted to the AC using a specific framework that will be provided to all PPs.

a) Data availability

Time Series choice. The availability of the analogous Time Series by EUROSTAT should always be checked. In case of double availability the choice should be based on the highest suitability for projections (length; frequency; territorial breakdown; etc.).

The situation concerning the declared availability of data for the construction of projections by sex and age of the Foreign Population, in the light of the evidence of the project document "Data Required to Construct Population Projections and Policy Scenarios Working Paper 2 Activity 3.3", can be briefly described in these terms:

[1] "International migration data as input for population projections" by A.Herm, & M.Poulain, in Work session on demographic projections, Lisbon, 28-30 April 2010, EUROSTAT Methodologies and Working Papers, pp. 255-268.

1) **Time Series Complete Availability (Nuts-2 or lower)**

There is availability of time series with appropriate length for the stocks of foreign population by gender / age, as well as for Fertility (TFR and / or specific), Mortality, Arrivals and departures by age /sex /nationality. In this case the projections, even if not available and / or published, may be easily calculated with the Cohort Component Model as described below.

2) Time Series Unavailability (Nuts-2 or lower)

In case of unavailability of time series necessary (but not sufficient) condition is the availability of census data or point data derived from surveys or other sources, at least for Foreign population by sex / age.

- If there are no tables of mortality at regional level for the foreign population but the mortality table is available at the national level you can take this as a national structure for the regional level. This can also be done for the other dimensions such as fertility and immigration. If you have at your disposal regional immigration figures, for example by sex, but the age structure is only available at the national level, we can assume the latter as a regional distribution in order to have an estimate of the regional population projection.

- The foreign population can alternatively be regarded as a subpopulation (see section 3 on Extended Population Projections). The incidence of foreigners (share or weight) at a particular time, such as the Census, can be applied to the projected total population. If the incidences (shares or weights) are specific (by sex / age) you can get a robust result. But this holds only in case of an hypothesis of constant incidences, then within the Scenario 2(see item g). Otherwise, the assumptions must be made on "diminishing share / weights" on the hypothesis NO Migration (closed population, see g))or increasing shares / weights in the case of migration flows increases (see item g).

- In the attempt of projecting foreign population, additional information and estimates on emigration rates and amounts, total fertility rates for population with foreign background, can be found in the Eurostat papers[2].

1. [2] EUROSTAT – Methodologies and Working Papers «Fewer, older and multicultural? Projections of EU population by foreign / national background» (2011) by G. Lanzieri. Eurostat Statistics in Focus – Population and Social Conditions 1/2010"Regional population projections EUROPOP2008:Most EU regions face older population profile in 2030" by K. GIANNAKOURIS. **van Wissen; van der Gaag; Rees; Stillwell (2008)** In search of a modeling strategy for projecting internal migration in European countries. Demographic versus economic-geographical approaches.

b) Definitions[3]

1) Population

Population on 1st January should be based on concept of *usual resident population*, and it is number of the inhabitants of a given area on 1 January of the year in question (or, in some cases, on 31 December of the previous year). The population figures can be based on data from the most recent census adjusted by the components of **population change** produced since the last census, or based on population registers.

Usually resident population means all persons having usual residence in a country at the reference time.

Usual residence means the place where a person normally spends the daily period of rest, regardless of temporary absences for purposes of recreation, holidays, visits to friends and relatives, business, medical treatment or religious pilgrimage.

The following persons alone are considered to be usual residents of the geographical area in question:

- those who have lived in their place of usual residence for a continuous period of at least 12 months before the reference time; or
- those who arrived in their place of usual residence during the 12 months before the reference time with the intention of staying there for at least one year.

Demographic balance - is the equation that describes the change in the size of the population due to the flows of live births, deaths, immigration and emigration that occur in the reference year T.

Population[T+1] = Population[T] + Births[T, T+1] - Deaths[T, T+1] + Net Migration*[T, T+1]

where:

Population[T+1] = total population on 1st January of the year T+1;

[3] Statistical concepts and definitions at http://epp.eurostat.ec.europa.eu/cache/ITY_SDDS/en/demoreg_esms.htm

Population[T] = total population on 1st January of the year T;

Births[T, T+1] = total number of live births in the time interval T to T+1;

Deaths[T, T+1] = total number of deaths in the time interval T to T+1;

Net Migration*[T, T+1] = net migration including statistical adjustment in the time interval T to T+1.

Population density - the ratio of the (annual average) population of a region to the (land) area of the region; total area (including inland waters) is used when land area is not available.

Crude rate of population change - the ratio of the total **population change** during the year to the **average population** of the area in question in that year. The value is expressed per 1000 inhabitants.

Crude rate of natural change - the ratio of natural change over a period to the average population of the area in question during that period. The value is expressed per 1000 inhabitants.

2) Mortality

Crude death rate - the ratio of the total number of deaths during the year to the average population in that year. The value is expressed per 1000 inhabitants.

Infant mortality rate - the ratio of the total number of deaths of children under one year of age during the year to the number of live births in that year. The value is expressed per 1000 live births.

Life expectancy at given exact age - the mean number of years still to be lived by a person who has reached a certain exact age, if subjected throughout the rest of his or her life to the current mortality conditions (age-specific probabilities of dying).

Life table is one of the most important and most widely used devices in demography, summarizing various aspects of the variation of mortality with age and showing, for each age, the probability that a person of that age will die before his next birthday. Functions pertaining to mortality are available in distinct tables: age specific death rates, probabilities of dying between exact ages, probability of surviving between exact ages, number left alive at a given exact age, number dying between exact ages, person-years lived between exact ages, total person-years lived above given exact age and life expectancy at given exact age. Eurostat uses the concept of age completed for the calculation of the mortality indicators by age.

3) Fertility

Crude birth rate - the ratio of the total number of **live births** during the year to the average population in that year. The value is expressed per 1000 inhabitants.

Total fertility rate (TFR) - the mean number of children that would be born alive to a woman during her lifetime if she were to pass through her childbearing years conforming to the fertility rates by age of a given year.

Fertility rates by mother's age (Age Specific Fertility Rate) - the number of births of mothers of age x to the average female population of age x.

Eurostat uses the concept of 'age completed' for calculation of the fertility indicators.

4) Migration

Net migration - the difference between the number of immigrants and the number emigrants from a given region during the year (net migration is therefore negative when the number of emigrants exceeds the number of immigrants).

Net migration including statistical adjustments - a general estimation of the net migration based on the difference between population change and natural change between two dates (in Eurostat database it is called *net migration plus statistical adjustment*). In different countries net migration including statistical adjustment may cover, besides the difference between inward and outward migration, other changes observed in the population figures between 1 January for two consecutive years which cannot be attributed to births, deaths, immigration or emigration.

Crude rate of net migration including statistical adjustment - the ratio of the net migration including statistical adjustment during the year to the average population in that year. The value is expressed per 1000 inhabitants. The crude rate of net migration is equal to the difference between the crude rate of population change and the crude rate of natural change (that is, net migration is considered as the part of population change not attributable to births and deaths). It is calculated in this way because immigration or emigration flows are either not available or the figures are not reliable.

5) Household

A 'private household' means "a person living alone or a group of people who live together in the same private dwelling and share expenditures, including the joint provision of the essentials of living". Refer to Eurostat for the different types of household.[4]

6) Level of education

International classification ISCED 1997 system is intended: Level 0 Pre-primary education, Level 1 Primary education, Level 2 Lower secondary education, Level 3 (Upper) secondary education, Level 4 Post-secondary non-tertiary education, Level 5 First stage of tertiary education, Level 6 Second stage of tertiary education

[4] See http://epp.eurostat.ec.europa.eu/cache/ITY_SDDS/en/ilc_esms.htm

c) Projections Method

The cohort-component method is the most commonly used technique to project future population size.

Population projections should be made for successive years running from the 1st of a the base year to the next using the cohort component method. Demographers who utilize the method could use a different interval length, to say the time between the launch year and target dates (common target dates are 2015 and 2020). Usually five year intervals are used. This occurs when most of the data needed to implement the cohort-component method available are classified in five-year.

The basic concept behind the cohort-component method is that behaviours that prevailed recently will continue in the future. This assumption is quite strong and often erroneous. However, the time-horizon for MMWD projections (up to 2020) is well suited to this assumption of the cohort-component method. It is in fact projections closer to the time horizon of the short-term (five years) that temporal horizon of the medium term (20 years).

The method is relatively standardized, however, variations in the application of the method are often needed to account for the (national, regional and sub-regional) type of data available for input.

In the cohort-component method three components of change migration, birth, and death, drive changes in population from one period to the next. Projections are made for age-gender groups as migration and mortality rates tend to differ significantly by age and sex. Similarly, birth rates depend on the age of the potential mother. For the cohort-component method, the three components are addressed individually then combined to project the population.

In summary you should:

- "To grow old" those who are already present in the population at the beginning of the forecast period (mortality);
- Predict the number of births in the meantime (fertility) and make them survive until the end of the forecast range (mortality);

- "Correct" the population with the number of inputs and outputs occurring over the forecast period (migration)

The C-CM calculation can be summarized as follows:

Annual Frequency

$\textbf{\textit{Population}}_{t} = \textbf{\textit{Population}}_{t-1} +$ $Births_t$ - $Deaths_t$ + In-$Migrants_t$ – Out-$Migrants_t$

For each age, the starting population plus net inward migrants less the number of deaths produces the number in the population, one year older, at the end of the year. Survivors of those born during the year are then added. Age is defined as completed years at the last birthday. Migration, deaths and births are all assumed to occur evenly throughout the year.

The procedure is carried out separately for the two sexes.

Five-years frequency

Assuming you have the structure and the amount of population at time t, at time t +5 the population of 5 years or more is the same as that already present at time t minus the deceased and emigrants, plus the immigrants . Therefore, each cohort of age x, x +4 must be adjusted on the basis of their mortality and migration.

The projection process ends with the estimation of newborns in the range t, t +5, which correct for the respective mortality and migration, will constitute the age group between 0 and 4 years old at time t +5.

The procedure is carried out separately for the two sexes.

The 1st January population estimates from each country are used as the starting population. The numbers of births, deaths and migrants are calculated using the assumptions of fertility, mortality and international migration.

Data required are the following:

- An initial population classified by sex and age;
- A table of mortality on the population and the period in question or adaptable to;

- A structure of fertility by age adaptable to the period considered; A structure of migration by sex and age valid for the forecast period

Cohort Component Method for Closed Population

Let us for simplicity in the following situation (see g))

- Closed population (no migration)

- Constant fertility and mortality.

It follows that the population already born in t is changed only for the effect of mortality.

If we set: q_x = probability of death; l_x = left alive; l_x= deaths; p_x = probability of survival; L_x = years lived, we can derive:

$d_x= l_x \times q$; $l_x= l_{x-1} – d_{x-1}$; $p_x=1-q_x$; $L_x = l_x - (1-\delta)\, d_x$

Then we consider the quantity determined from a mortality table called Probability of perspective survival : $S_{x,x+4} = L_{x+5,x+9}/L_{x,x+4}$

This quantity represents the probability that, at time t, people of age x, x +4 shall survive until time t +5 (when they will have x +5, x +9 years).

We therefore have:

$$P_{x+5,x+9}(t+5) = P_{x,x+4}(t) \cdot s_{x,x+4}\,, x = 0,5,10,15,\ldots, 70$$

The mathematical relationship is slightly different for the last age group. In fact, both $P_{75/79}$ then P_{80+} at time t are included in the class P_{80+} at time t +5. So:

$$P_{80+}(t+5) = [P_{75,79}(t) + P_{80+}(t)] \cdot s_{75+}$$

The first age class in t +5 will be made up of all those born in the interval t, t +5 correct for the respective mortality:

$$P_{0,4}(t+5) = N(t, t+5) \cdot s_{Born}$$

where S_{Born} x is the prospective probability of survival of newborns.

The last step is about the calculation of the amount of births *N (t, t +5).* The estimate of the number of births is obtained by considering that:

$$f_{x,x+4} = \frac{N_{x,x+4}(t,t+5)/5}{\frac{1}{2}\left(P^f_{x,x+4}(t) + P^f_{x,x+4}(t+5)\right)},$$

So the numerator is divided by 5 to keep the annual size of the specific rate. In fact, the number of births in the numerator is recorded in five calendar years.

So those born of women aged x, x +4 are:

$$N_{x,x+4}(t,t+5)=5\cdot f_{x,x+4}\cdot\tfrac{1}{2}\left(P^{f}_{x,x+4}(t)+P^{f}_{x,x+4}(t+5)\right)$$

The number of total births will be given by the sum of births had from the total of fertile women:

$$N(t,t+5)=\sum_{x=15,19}^{45,49}N_{x,x+4}(t,t+5)$$

Once you have determined the total number of births in the five years, births must be distributed between the sexes that is, identify how many female and male births.

Because the sex ratio at birth is constant in each population and equal to 106 males per 100 females, then out of 100 births you get 51.5 males and 48.5 females, then the number of total births has to get multiplied to these percentages and get the male and female births.

A cycle of five-year projection can be repeated in order to obtain a projection in the longer term.

In summary, each cycle of projection (t, t +5) requires the following steps:

A) Identification of the basic elements

1. Population at time t by sex and age
2. Model of survival by sex and age
3. Specific rates of fertility

B) Calculation of surviving for those already born at time t
Application of the prospective probabilities of survival of individuals already present in the starting population

C) Estimated number of births provided for each sex

D) Calculation of survivors among the new born

Cohort Component Method with Migration

First, it is appropriate to assess separately the immigration from emigration. it is useful to observe the differences between the population under study and the other in the same area, in terms of demographic and economic development.

To take account of migration, we need to modify the formulas adopted for the projections to take into account both net migration and the mortality of migrants.

Considering the net migration (NM) in a given time interval (t, t +1), and assuming that the probabilities of death during this period are similar to the indigenous population and migrants, we have:

$$P_{x+1}(t+1) = P_x(t) + NM(t,t+1) - \left[P_x(t) + \tfrac{1}{2}NM(t,t+1)\right] \cdot q_{x,x+1}$$

Then, at the time t +1 you should subtract the dead from the indigenous population at time t and net migration recorded in the interval t, t +1,.

Note that the chances of dying applies only to half of the net migration since it is assumed that both the emigrants that immigrants remain on average present, in the population studied, for only half of the time between t and t +1.

Performing simple calculations we arrive at the following relationship:

$$P_{x+1}(t+1) = P_x(t) \cdot s_x + \tfrac{1}{2}NM(t,t+1) \cdot (1+s_x)$$

For new born we have the following equation:

$$P_0(t+1) = N(t,t+1) \cdot s_{Born} + \tfrac{1}{2}NM_{Born}(t,t+1) \cdot (1+s_{Born})$$

where NM_{Born} is the balance between children born to immigrants and new born emigrants.

If entering the migratory component is technically easy to solve, not just as simple is the prediction of the amount of migration and, even more, their specification by sex and age.

Generally you make a prediction on the flows in absolute terms, for example by assuming the value of net migration for each forecast period and then correcting these values with the prediction equations.

d) Geographical Breakdown

The territorial level for the development of population projections is to NUTS-2, but projections can also be developed to the lower level (NUTS3 and LAU1).

National Consistency. As the MMWD project focus on Nuts-2 and Nuts-3, in the process of projections construction the components of population change (births, deaths and migration) should not be constrained to the national level.

e) Starting point of the Projections

The starting point for the projections is the base population. This is taken as the 1st of January of the year in question or the 31st December of the previous year's population estimate from each country.

f) Projection Extension

Projections should extend from 1st January of 2013 or 31st of December 2012 to 1st of January of 2020 or 31st of December 2019.

g) Projections Frequency

Annual

The projections should be one-year age group, sex and national/non national.

Project Partners can project and/or aggregate by year and / or select some points in time (such as the common years 2015 and year 2020[5]) or other according to their needs.

The use of five-year classes can have the advantage, in the case of projections on level NUTS-4 or LAU1, to avoid the risk of data gaps or very irregular shapes (e.g.: the more the area is small, the

[5] EUROPOP 2010 projections are 5 years over five years, starting from 2015.

easier it is to find, even on an average of years, no born to a mother 35 years old or no deaths for the same age). However, one must keep in mind that this means to evolve in five-years steps so, if you want to have at least two common projected points (2015 and 2020), it will be necessary to start from the base population at the 1st of January 2010 or the 31st of December 2009. In the case of evolution with annual pace you can use any year.

h) Scenarios

Population scenarios will be made according to the three scenarios mentioned in the ESPON's DEMIFER project. Additional scenarios could also be studied by each PP in the attempt to capture specific issues and flows, assuming specific and more detailed hypothesis depending on the migration profile of the territory.

The disadvantage of an analytical method as the Cohort-Component Method is above all not to have information on the degree of reliability of the projection obtained (eg, confidence intervals). One way to overcome this limitation is to resort to the presentation of a range of results (scenarios). From their comparison can be grasped the importance of the individual components involved.

Migration in the projections timeframe (2013-2020) is the only component that can show a significant change. Birth and mortality remain stable as very limited changes could occur in the reproductive behaviours and mortality trends.

Regarding migration, interval sustainability of the hypotheses is usually very short (no more than 10 years) because migration flows can vary very quickly (eg as a result of particular laws). This constraint, however, suits to the time horizon considered.

1. The **FIRST** scenario is a scenario with zero (net) migration or, otherwise said "closed population" scenario.

The hypothesis of the absence of migration is of course unlikely. It would be also the hypothesis of no or negligible net migration, as in the reference area of the project MMWD are prevalent areas of origin or destination of the flows. However, this choice may be useful to assess what would be the evolution of the population as a function of only the natural component.

2. The **SECOND** scenario assumes constant migration (periodical average for the last 10 years taking into account migration peaks to regularize historical series).

3. The **THIRD** scenario considers the average migration trends for the period 2009-2012 (or last available data) in order to isolate the effects of the economic crisis on population movements.

3. Extended Population Projections

The extended population projections are:

- Projection by age/sex/households
- Projection by age/sex/education attainment

Starting from the forecasts of population, we can obtain estimates of other subpopulations, in the case of the project MMWD families and sub-populations for education attainment.

Static Approach

Specific rates are applied to the projected population without taking into account the internal dynamics of the phenomenon studied. We limit ourselves to observe the final results in terms of stock.

For example, the prediction of the amount of households does not take into account the dynamics associated with family behavior (divorce, cohabitation, remarriage, etc..). Hence, the static approach will give satisfactory results only if there are rapid changes and of opposite sign for different items.

Dynamic Approach

It consists in managing complex models capable of controlling a priori information and assumptions on individual factors capable of influencing the dynamics.

The choice between the two methods depends, besides the characteristics of the aggregate object of interest, from: a) the structure of the basic predictions; b) the availability of data; c)the degree of detail that you want to achieve.

- **Households Projections**

Among the forecasting models on families one of the methods used is that based on the " household's headship " rate. Projections give out estimation of the number of families, families by family size and type of family.

The "household's headship" rate

The "household's headship" rate is the ratio between the number of head of household by sex and age group and the total population of the same sex and age group.

Data Requirement

- Projected population
- Households type by age and sex of the head of household
- Households by household size and type

For the calculation of these rates is therefore necessary to know the number of families whose breadwinner is in a certain age group and sex .

The number of households expected in a given time t is obtained by multiplying the projected population by age and sex at time t for the corresponding "household's headship" rates.

In order to obtain the distribution of households by number of members, households are distributed by number of members using the census data.

Distributions should be modified to achieve equality between the population predicted by the demographic model and the population resulting from the distribution of households by number of members. Should be mentioned that typically base population projection considers the total usual resident population while a part of that did not leave in a private household , namely those people living in collective house. This situation imply that, before applying headship rate, the population by age and sex should be divided in living or not living in private household and headship rate should be applied only to the first group.

- **Projections of Population by educational attainment**

If you choose the static approach, considering the projection of sub-populations for educational attainment, to project the population of individuals with "trait γ", the ratio of individuals with the trait by age and gender to the total population of the sex-age group are calculated.

To project the population, this rate is then multiplied by the projected population of that age-sex group calculated previously. The total number of individuals projected with "trait γ" is found by summing over age and sex.

In a dynamic perspective, as suggested by Region Emilia Romagna[6], the methodology proposed by Goujon (2006)[7] and Barakat, Goujon, Skirbekk, Sanderson & Lutz (2010)[8] appear feasible.

The core of the methodology are a*ge- and sex-specific education transitional probabilities* along cohort line.

- They represent age-specific transition probabilities for men and women to move from one educational attainment level ($status_l$) to another ($status_{i+1}$). Educational levels are hierarchical, so an individual can not enroll to a given level of education if he/she has not completed all the previous levels.

- A pattern of age-specific transition probabilities for cohorts can be interpreted as a probability for a population at a specific age and in a given time t to move to an higher category of educational attainment by the time t+1.

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6 "Contribution of the Emilia-Romagna to the definition of a common reference methodology for population projections – basic and extended. Act.4.2" by A,. Mazzocchetti, MMWD Meeting, September 19,2013.
7 "Report on projections by level of education (Future human capital: Estimates and projections of education transition probabilities)" by A. Gouion, Vienna Institute of Demography, Austrian Academy of Sciences http://www.nidi.nl/Content/NIDI/output/micmac/micmac-d3.pdf.
8 "Projection of populations by level of educational attainment, age, and sex for 120 countries for 2005-2050" by KC Samir, B. Barakat, A. Goujon, V. Skirbekk, W. Sanderson & W. Lutz, *DEMOGRAPHIC RESEARCH* VOLUME 22, ARTICLE 15, pages 383-472.
http://www.demographic-research.org/Volumes/Vol22/15/ DOI: 10.4054/DemRes.2010.22.15

- People complete a level of education at different ages because of differences in age at entrance and repetition of grades. Transition probability to a given educational level are therefore not zero for a limited range of different ages.

Data Requirement

- Projected population by age and sex
- Population Time Series by age, sex and levels of educational attainment

Assumptions

- Starting age: 5 years
- Upper age level: usually fixed at 30 as adult education is very limited and has a negligible impact on the distribution of the population by educational level.
- No mortality and no migration effect
- Trend of transition probabilities

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Appendix

Mortality

To project the number of deaths for each age-gender group. Here the number of surviving individuals ${}_nSURVP^g_{x,l}$ is equal to the number of residents of gender g and age x to x + n at time l, ${}_nP^g_{x,l}$, times the survival rate for that age-sex combination ${}_nS^g_x$.

$${}_nSURVP^g_{x,l} = {}_nP^g_{x,l} \times {}_nS^g_x$$

Migration

In- and out-migration rates are calculated for each age-gender combination using equations (1) and (2). The in-migration rate for individuals of gender g age x to x + n during the period l-t and l is found by dividing the number of individuals of gender g age x + z to x + z + n that move into the area during period l-t to l by the number of individuals of gender g age x to x + n that live outside the geographic region of interest at the beginning of the period, that is l-t. This value is typically expressed as the difference between the total population of the United States of gender g age x to x +n, in year l-t less that residing in the region of interest.

$${}_nINMIGRATE^g_{x,l-t \text{ to } l} = \frac{{}_nINMIG^g_{x+z,l-t \text{ to } l}}{\left({}_nUSP^g_{x,l-t} - {}_nP^g_{x,l-t}\right)} \quad \text{.............(1)}$$

The calculation of the out-migration rate is similar, with the exception that the population of the community of interest is the denominator (2).

$${}_nOUTMIGRATE^g_{x,l-t \text{ to } l} = \frac{{}_nOUTMIG^g_{x+z,l-t \text{ to } l}}{{}_nP^g_{x,l-t}} \quad \text{.............(2)}$$

To project in-migration between the launch year, l, and the target date, t, the rate calculated in equation (1) is multiplied by the population of the United States less that of the community in the launch year (3) for each age-gender class. Similarly, out-migration is projected by multiplying the out-migration rate from (2) times the population by age-gender group at year l (4).

$${}_{n}INMIG^{g}_{x+z,l \text{ to } t} = {}_{n}INMIGRATE^{g}_{x,l \text{ to } t} \times \left({}_{n}USP^{g}_{x,l} - {}_{n}P^{g}_{x,l} \right) \text{.............(3)}$$

$${}_{n}OUTMIG^{g}_{x+z,l \text{ to } t} = {}_{n}OUTMIGRATE^{g}_{x,l \text{ to } t} \times {}_{n}P^{g}_{x,l} \text{.............(4)}$$

.............(5)

Natality

Next, the number of births for the region is projected. The first step in this process is to determine the age-specific birth rate, which is traditionally noted as births per 1,000 women in their childbearing years (ASBR). This is done by dividing the number of births born to a mother age x to x + n, ${}_{n}B_{x}$, divided by the total number of women in that age group, ${}_{n}FP_{x}$, and then multiplying that number by one thousand as shown in equation (6).

$${}_{n}ASBR_{x,t} = \left({}_{n}B_{x} \big/ {}_{n}FP_{x} \right) 1000 \text{.............(6)}$$

To accommodate the fact that half of the women in an age group at the beginning of a period will pass into the next age group, the adjusted age-specific birth rate is needed. This is calculated by finding the arithmetic average of successive age-specific birth rates as shown in equation (7).

$${}_{n}ADJASBR_{x,t} = ({}_{n}ASBR_{x,t} + {}_{n}ASBR_{x+z,t}) / 2 \text{.............(7)}$$

The number of women of childbearing age, referred to as the at-risk female population, is calculated using (8). This value is equal to the female population in the launch year plus in-migrants, less the number projected to out-migrate or die. It is assumed that the women who die live for half of the interval.

$${}_{n}ATRISK_{x,t} = {}_{n}P^{f}_{x,l} - .5 \times {}_{n}D^{f}_{x,l \text{ to } t} + {}_{n}INMIG^{f}_{x,l \text{ to } t} - {}_{n}OUTMIG^{f}_{x,l \text{ to } t} \text{.............(8)}$$

The projected number of births between the launch and target years to mothers age x to x +n is equal to the adjusted age-specific birthrate times the at-risk female population divided by one thousand (9). The total number of births is found by summing across all ages of mothers (10).

$$ {}_nB_{x,l \text{ to } t} = {}_nADJASBR_{x,t} \times {}_nATRISK_{x,t} / 1000 \quad \text{.............(9)} $$

$$ B_{l \text{ to } t} = \sum_{Ages} {}_nB_{x,l \text{ to } t} \quad \text{.............(10)} $$

The total number of births, B_l to $_l$, is multiplied by rate of male births, *PCTM*, to determine the number of males born in the period, MB_l to $_l$ (11). Similarly, the number of births is multiplied by 1 minus the male birth rate to determine the number of females born, FB_l to $_l$(12).

$$ MB_{l \text{ to } t} = B_{l \text{ to } t} \times PCTM \quad \text{.............(11)} $$

$$ FB_{l \text{ to } t} = B_{l \text{ to } t} \times (1 - PCTM) \quad \text{.............(12)} $$

These values are then multiplied by the gender-specific survival rate to project the number of male, ${}_nM_{0,z}$, and female, ${}_nF_{0,z}$, residents age zero to z in the geographic region (13,14).

$$ {}_nM_{0,z} = MB_{l \text{ to } t} \times {}_nMS_0 \quad \text{.............(13)} $$

$$ {}_nF_{0,z} = FB_{l \text{ to } t} \times {}_nFS_0 \quad \text{.............(14)} $$

Combining the Components

The determination of the projected value of the remaining age-sex groups is done by adding the surviving population with the number of in-migrants less the number of out-migrants as shown in (15).

$$ {}_nP^g_{x+z,l \text{ to } t} = {}_nSURVP^g_{x+z,l \text{ to } t} + {}_nINMIG^g_{x+z,l \text{ to } t} - {}_nOUTMIG^g_{x+z,l \text{ to } t} \quad \text{.............(15)} $$

Projecting Special Populations

To project the population of individuals with trait γ, the ratio of individuals with the trait by age and gender to the total population of the gender-age group are calculated (16). To project the population, this rate is then multiplied by the projected population of that age-gender group calculated previously. The total number of individuals projected with trait γ is found by summing over age and gender (17).

$$ {}_n\hat{p}^{g,\gamma}_x = {}_nP^{g,\gamma}_{x,l} / {}_nP^g_{x,l} \quad \text{.............(16)} $$

$$ {}_{n}P_{x,t}^{\gamma} = \sum_{g} \sum_{ages} {}_{n}\hat{p}_{x}^{g,\gamma} \times {}_{n}P_{x+z,t}^{g} \quad \text{.............(17)} $$

To project the number of households with trait γ, it is further assumed that the household size will remain the same across time, a strong assumption. Household size is calculated by dividing the launch population by the number of households in the launch year (18). To project the number of households in the target year, the projected population in the target year is divided by this rate (19). Equations (20) and (21) project the number of households with trait γ in the same manner as (16) and (17).

$$ \hat{H} = P_l / H_l \quad \text{.............(18)} $$

$$ H_t = \frac{P_t}{\hat{H}} \quad \text{.............(19)} $$

$$ \hat{H}^{\gamma} = H_l^{\gamma} / H_l \quad \text{.............(20)} $$

$$ H_t^{\gamma} = \hat{H}^{\gamma} \times H_t \quad \text{.............(21)} $$